

Multiplying Fractions and Mixed Numbers - Black Problems

1. Find a value of x so that $3\frac{1}{2} \cdot \frac{1}{x} + \frac{5}{3x}$ is equal to 1.
2. One-fourth of Holttown High School's students are seniors, one-third are juniors, and the other 300 students are sophomores. Of the seniors, two-fifths are boys. How many senior girls are students at Holttown High School?
3. Tony the Tortoise walked 10 inches during the first hour of his journey. He walked one and one-half times that distance during the second hour, and in general during the n^{th} hour he walked $\left(1 + \frac{1}{n}\right)$ times the distance walked during the $n - 1^{\text{st}}$ hour. How many inches did Tony walk during the first three hours of his journey?
4. In a study of the effects of time pressure on Mathletes, one-third of the Mathletes were given buzzers and the rest were not. All of the Mathletes answered a particular question. One-third of those with buzzers answered the question correctly, while one-fourth of those answering the question correctly had buzzers. What fraction of the students without buzzers answered the question correctly? Express your answer as a common fraction.
5. The fourth Farey sequence, $F_4 = \frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$, is the list, written in increasing order, of all the common fractions with distinct values from 0 through 1, inclusive, that use only the integers 0, 1, 2, 3 or 4 as numerators and denominators. In the fifth Farey sequence, what is the product of the third and tenth terms? Express your answer as a common fraction.

The Most Appealing Nose Contest

As a service project, the Student Council at my school wanted to raise money for the Humane Society. After a brief debate, the students decided to hold a *Most Appealing Nose* contest. Each contestant would submit a photograph of his or her pet, along with the application fee of \$5.

Half of the photographs submitted were pictures of cats, so it was no surprise when a lovely black cat named Munin won the top prize, \$25 worth of cat food and toys. A quarter of the photographs received were pictures of dogs, $\frac{1}{8}$ were pictures of horses, $\frac{1}{16}$ were pictures of fish, and 13 were gerbils. Only $\frac{1}{32}$ of the photos were pictures of canaries, and some students were surprised when Chandlor Canary, belonging to Principal Peters, flew away with the second place prize, a \$1.79 bag of birdseed.

6. If the only expense was the money spent for prizes, how much money did the Student Council raise for the Humane Society?

7. Milk Madness

Georgina loves milk, and she also loves working with numbers. Just for fun she thought of combining a number pattern and the drinking of a gallon of milk. She imagined asking someone to do this:

Start by drinking one-half a gallon, then two-thirds of what is left, then three-fourths of what is left, and then four-fifths of what is left, and so on.

She thought how cool it would be to figure out exactly how much milk would be left!

How much milk did she figure would be left after taking **ten** drinks?

8. If b is positive, what is the value of b in the geometric sequence $9, a, 4, b$? Express your answer as a common fraction.
9. From an 1855 text, School Arithmetic, by Charles Davies: "If twenty grains make one scruple; three scruples make one dram; eight drams make one ounce; and twelve ounces make one pound; what part of an ounce is $\frac{3}{10}$ of a scruple?" Express your answer as a common fraction.

Multiplying Fractions - Black Solutions

1. $\frac{31}{6}$
2. If $\frac{1}{4}$ of Holttown High School are seniors and $\frac{1}{3}$ are juniors, then together they account for $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$ of the school. The 300 sophomores account for the other $\frac{5}{12}$ of the school, so there must be $\frac{12}{5} \times 300 = 720$ students in school. The senior class must have $\frac{1}{4} \times 720 = 180$ students. If $\frac{2}{5}$ of these are boys, then there must be $\frac{3}{5} \times 180 = \mathbf{108 \text{ girls}}$.
3. Tony the Tortoise walked 10 inches the first hour, $10 \times \frac{3}{2} = 15$ inches the second hour and $15 \times \frac{4}{3} = 20$ inches the third hour. The total distance in the first three hours was $10 + 15 + 20 = \mathbf{45}$ inches.
4. Let's pick a nice number of Mathletes to see if we can make this work. Since we will be taking a third of the number and then taking a third of that new number, let's make our number of Mathletes a multiple of 9. Suppose there are 36 Mathletes. Then 12 have buzzers and 24 do not. If one-third of those with buzzers got the question right, then 4 Mathletes with buzzers answered correctly, and 8 Mathletes with buzzers did not answer correctly. If the 4 Mathletes with buzzers who answered correctly represent $\frac{1}{4}$ of those who answered correctly, then there must have been 16 who answered correctly. That means 12 of those who answered correctly did not have buzzers. That's $\frac{1}{2}$ of the 24 Mathletes without buzzers.
5. The fifth Farey sequence is $\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}$. The third term is $\frac{1}{4}$ and the tenth term is $\frac{4}{5}$, so the desired product is $\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$.
6. **The Most Appealing Nose Contest.** The Student Council raised \$2,053.21 for the Humane Society. For this problem, I drew a circle and divided it into 32 pieces, because $\frac{1}{32}$ of the pictures were of canaries. So, I then shaded in half of the circle and labeled it CATS, because one half of the pictures were of cats. Then, I shaded $\frac{1}{4}$ of it, and labeled it DOGS, because one-fourth of the pictures were of dogs. Since $\frac{1}{8}$ of the pictures were of horses, I then shaded $\frac{1}{8}$ of the pie, and labeled it HORSES. Now $\frac{1}{16}$ of the pictures are of fish, so I shaded in $\frac{1}{16}$ of the pie and

labeled it. Only $\frac{1}{32}$ of the pictures were canaries, so I shaded in that $\frac{1}{32}$ and labeled it. Since there was only one more wedge, and one more animal (gerbils), I shaded that in for Gerbils. Now, since there are 13 Gerbils, and the canary wedge is the same size as the gerbils, I saw that there are 13 canary pictures. Now the fish wedge is twice as big, so I multiplied 13 by 2. I got 26. So there 52 horse pictures submitted. Dogs is twice as big, so I multiplied 52 by 2, and I got 104. Cats is twice as big, so I multiplied 104 by 2 and got 208. I then added all of the numbers and got 416. Since the application fee is \$5.00, I multiplied 416 pictures by \$5.00. I got a total of \$2,080. So to see how much money they earned, I subtracted the total of money spent for prizes, which was \$26.97. I got the total for the prizes by adding the \$25.00 grand prize, and the \$1.79 second prize. I subtracted the \$26.97 from \$2,080.00 and got \$2,053.21. So, \$2,053.21 is the amount the council earned for the Humane Society.

7. There would be $\frac{1}{39,916,800}$ of a gallon left after ten drinks. That is the same as .00009 cubic millimeters.

I noticed that you have 1 gallon and you take $\frac{1}{2}$ away from it you have $\frac{1}{2}$ left. If you think of $\frac{1}{2}$ gallon as 1 gallon and take $\frac{2}{3}$ away you'll seem to have $\frac{1}{3}$ gallon. You actually have $\frac{1}{3}$ of $\frac{1}{2}$ gallon or $\frac{1}{2} * \frac{1}{3}$ gallon. You keep multiplying like that and get:

$$\frac{1}{2} * \frac{1}{3} * \frac{1}{4} * \frac{1}{5} * \frac{1}{6} * \frac{1}{7} * \frac{1}{8} * \frac{1}{9} * \frac{1}{10} * \dots$$

Wait! We don't know where to stop. I noticed that where the drink in the order is one more than the n in $\frac{1}{n}$ left of whatever you had before. So, on the tenth drink, the fraction would be $\frac{1}{11}$. Now we can finish the equation:

$$\frac{1}{2} * \frac{1}{3} * \frac{1}{4} * \frac{1}{5} * \frac{1}{6} * \frac{1}{7} * \frac{1}{8} * \frac{1}{9} * \frac{1}{10} * \frac{1}{11} = \frac{1}{39,916,800}.$$

8. To find the next terms in a geometric series, you always multiply by the same factor. If the factor is greater than one, the terms get bigger and bigger. If the factor is less than one (but greater than zero), the terms get smaller and smaller. Moving from 9 to a to 4, we must multiply by the same factor twice. Let's call the factor x . We have $9x^2 = 4$. This means that $x^2 = \frac{4}{9}$ and $x = \frac{2}{3}$. Multiplying 4 by $\frac{2}{3}$,

we find that the value of b is $\frac{8}{3}$. (If we used the solution $x = -\frac{2}{3}$, then b would not be positive.)

9. $\frac{3}{10}$ scruple $\times \frac{1 \text{ dram}}{3 \text{ scruple}} \times \frac{1 \text{ ounce}}{8 \text{ dram}} = \frac{1}{80}$ of an ounce.